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Teleportation of a Pure EPR State via GHZ-like State

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Abstract This study proposes a novel teleportation using the GHZ-like state $\frac{1}{2}(|001\rangle + |010\rangle + |100\rangle + |111\rangle)$, in which a pure EPR state $\alpha|01\rangle + \beta|10\rangle$ can be perfectly teleported. Furthermore, the teleportation scheme is applied to construct a quantum secret state sharing (QSSS) protocol.

Keywords Teleportation · GHZ-like state · EPR state · Unitary operation

1 Introduction

The study of quantum information theory is an extended research in recent years. In the quantum information theory, entanglement is a key physical property and has been employed in many applications, e.g., quantum teleportation [1, 2, 7, 9, 10, 12, 13, 16], quantum key distribution [5], quantum secret sharing [3, 8, 14, 15, 17, 18] and etc. Teleportation is used to transmit an unknown quantum state to a receiver through the correlation of the entanglement and some auxiliary classical communications. Bennett et al. [2] proposed the first teleportation scheme. Subsequently, a large number of quantum teleportation schemes have been proposed by using the maximally entangled states (e.g. Einstein-Podolsky-Rosen (EPR) state and Greenberger-Horne-Zeilinger (GHZ) state) in [7, 10, 16] and other entangled states (e.g. symmetric W state and asymmetric W state) in [1, 9, 12, 13].

Recently, Yang et al. [13] used a GHZ-like state $|\phi_G\rangle = \frac{1}{2}(|100\rangle + |010\rangle + |001\rangle + |111\rangle)$ to teleport an unknown single quantum state $\alpha|0\rangle + \beta|1\rangle$. Unfortunately, in [13], a method to teleport an EPR state $\alpha|01\rangle + \beta|10\rangle$ using the GHZ-like state has not yet been proposed. Therefore, the aim of this study is to propose a teleportation scheme for an unknown pure EPR state via the GHZ-like state.

According to [3], a teleportation scheme can also be applied to establish a quantum secret state sharing (QSSS) protocol. In a secret sharing protocol, a secret can be divided into

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several parts for several agents, one for each respectively. Since each of the part reveals no information about the secret, any agent cannot obtain any information of the secret except that they cooperate together to do so. QSSS protocols have also been presented in [3, 8, 14, 15] to share quantum states. Based on the proposed teleportation scheme, this paper also tries to propose a QSSS protocol.

The rest of this paper is organized as follows. The next section introduces the proposed teleportation scheme. A QSSS protocol is described in Sect. 3. Finally, a short conclusion is given in Sect. 4.

2 Teleportation of an EPR State via GHZ-like State

In the section, we briefly describe the construction of the GHZ-like state. Then, a new unitary operation called U_S is defined here to facilitate a novel teleportation scheme, in which a pure EPR state can be teleported.

2.1 Review of GHZ-like State

Tripartite entanglement states are classified in [4] into two classes: GHZ class and W class. According to this concept of classification, the quantum state of $|\phi_G\rangle = \frac{1}{2}(|001\rangle + |010\rangle + |100\rangle + |111\rangle)$ belonging to GHZ class is called the GHZ-like state in [13]. The GHZ-like state can be constructed by using the EPR state and a single photon as follows:

Step 1. Prepare a single photon $|\varphi\rangle_1 = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)_1$ and an EPR state $|\phi_{EPR}\rangle_{23} = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)_{23}$, and then a tensor product is performed as follows:

$$\begin{aligned}|\phi_{G_0}\rangle_{123} &= |\varphi\rangle_1 \otimes |\phi_{EPR}\rangle_{23} \\ &= \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \otimes \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)_{23} \\ &= \frac{1}{2}(|001\rangle + |010\rangle + |110\rangle + |101\rangle)_{123}. \qquad (1)\end{aligned}$$

Setp 2. A controlled-not gate [11] is performed on $|\phi_{G_0}\rangle_{123}$, where the first particle is the control qubit and the second particle is the target qubit. The outcome can be presented as follows:

$$|\phi_G\rangle_{123} = \frac{1}{2}(|001\rangle + |010\rangle + |100\rangle + |111\rangle)_{123}. \qquad (2)$$

Experimentally, the EPR state and controlled-not gate can be implemented in practice. Therefore, the GHZ-like state will also be practically constructed.

2.2 The Proposed Teleportation Scheme

Let q_i denote the ith particle of GHZ-like state $|\phi_G\rangle_{123}$. Suppose that a sender (called Alice) holds the first particle q_1 of the GHZ-like state and the remaining particles q_2 and q_3 are owned by a receiver (called Bob). The pure EPR state that Alice wants to teleport is $|\phi_{EPR}\rangle_{ab} = \alpha|01\rangle_{ab} + \beta|10\rangle_{ab}$. The steps of the proposed teleportation scheme are described as follows (also shown in Fig. 1):

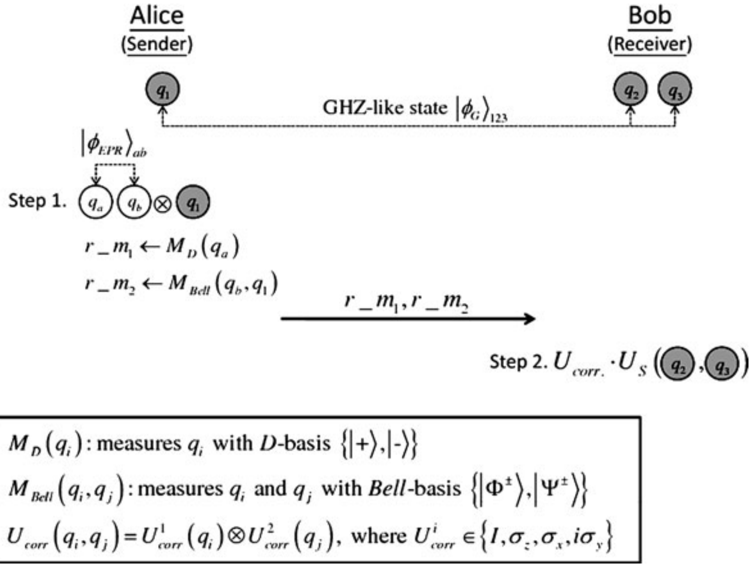

Fig. 1 The teleportation scheme of a pure EPR state

Step 1. Alice performs tensor product on q_b and q_1, and the state of $|\phi_{EPR}\rangle_{ab} \otimes |\phi_G\rangle_{123}$ can be shown as

$$\frac{1}{2}(\alpha|01100\rangle + \alpha|01010\rangle + \alpha|01001\rangle + \alpha|01111\rangle + \beta|10100\rangle + \beta|01010\rangle + \beta|01001\rangle + \beta|01111\rangle)_{ab123}. \tag{3}$$

Then, she measures q_a by the computational basis $\{|+\rangle, |-\rangle\}$ (called D-basis for short), and performs the Bell-measurement on q_b and q_1, where q_a and q_b denote the first and second particles of $|\phi_{EPR}\rangle_{ab}$, respectively. The measuring results are denoted as r_m_1 and r_m_2.

In order to present the process of measurement clearly, (3) can be separated into two parts:

$$\begin{aligned}
&\frac{1}{2\sqrt{2}}|+\rangle_a \otimes (\alpha|1100\rangle + \alpha|1010\rangle + \alpha|1001\rangle + \alpha|1111\rangle \\
&\quad + \beta|0100\rangle + \beta|0010\rangle + \beta|0001\rangle + \beta|0111\rangle)_{b123} \\
&= \frac{1}{4}|+\rangle_a \otimes \{\Phi^+_{b1} \otimes (\alpha|00\rangle + \alpha|11\rangle + \beta|10\rangle + \beta|01\rangle)_{23} \\
&\quad + \Phi^-_{b1} \otimes (-\alpha|00\rangle - \alpha|11\rangle + \beta|10\rangle + \beta|01\rangle)_{23} \\
&\quad + \Psi^+_{b1} \otimes (\beta|00\rangle + \beta|11\rangle + \alpha|10\rangle + \alpha|01\rangle)_{23} \\
&\quad + \Psi^-_{b1} \otimes (\beta|00\rangle + \beta|11\rangle - \alpha|10\rangle - \alpha|01\rangle)_{23}\}
\end{aligned} \tag{4}$$

and

$$\frac{1}{2\sqrt{2}}|-\rangle_a \otimes (\alpha|1100\rangle + \alpha|1010\rangle + \alpha|1001\rangle + \alpha|1111\rangle$$

$$
\begin{aligned}
&-\beta|0100\rangle-\beta|0010\rangle-\beta|0001\rangle-\beta|0111\rangle)_{b123}\\
&=\frac{1}{4}|-\rangle_a\otimes\{\Phi^+_{b1}\otimes(\alpha|00\rangle+\alpha|11\rangle-\beta|10\rangle-\beta|01\rangle)_{23}\\
&-\Phi^-_{b1}\otimes(\alpha|00\rangle+\alpha|11\rangle+\beta|10\rangle+\beta|01\rangle)_{23}\\
&+\Psi^+_{b1}\otimes(-\beta|00\rangle-\beta|11\rangle+\alpha|10\rangle+\alpha|01\rangle)_{23}\\
&-\Psi^-_{b1}\otimes(\beta|00\rangle+\beta|11\rangle+\alpha|10\rangle+\alpha|01\rangle)_{23}\},
\end{aligned}
\tag{5}
$$

where $\Phi^{\pm}_{b1}=\frac{1}{\sqrt{2}}(|00\rangle\pm|11\rangle)_{b1}$ and $\Psi^{\pm}_{b1}=\frac{1}{\sqrt{2}}(|01\rangle\pm|10\rangle)_{b1}$ are Bell states. Finally, the r_m_1 and r_m_2 are sent from Alice to Bob.

Step 2. Observing (4) and (5), we can find out that the state of $|\phi\rangle_{23}$ is a product state, but not a pure EPR state. Bob cannot immediately reconstruct the pure EPR state by using the four Pauli unitary operations $\{I,\sigma_z,\sigma_x,i\sigma_y\}$, where $I=|0\rangle\langle 0|+|1\rangle\langle 1|$, $\sigma_z=|0\rangle\langle 0|-|1\rangle\langle 1|$, $\sigma_x=|1\rangle\langle 0|+|0\rangle\langle 1|$, and $i\sigma_y=|0\rangle\langle 1|-|1\rangle\langle 0|$. In order to overcome this problem, a specific unitary operation U_S is defined as

$$
U_S=\frac{1}{\sqrt{2}}\begin{pmatrix}1&0&0&1\\0&1&-1&0\\1&0&0&-1\\0&1&1&0\end{pmatrix}.
\tag{6}
$$

The unitary operation U_S is used to transform the product state of $|\phi\rangle_{23}$ into a pure EPR state. For example, performing U_S on

$$
|\phi\rangle_{23}=\frac{1}{2}(\alpha|00\rangle+\alpha|11\rangle+\beta|10\rangle+\beta|01\rangle)_{23},
$$

the state of $U_S\cdot|\phi\rangle_{23}$ will become $\alpha|00\rangle_{23}+\beta|11\rangle_{23}$. After performing the U_S unitary operation on the (q_2,q_3), (4) and (5) are respectively transformed to

$$
\begin{aligned}
&\frac{1}{2\sqrt{2}}|+\rangle_a\otimes\{\Phi^+_{b1}\otimes(\alpha|00\rangle+\beta|11\rangle)_{23}+\Phi^-_{b1}\otimes(-\alpha|00\rangle+\beta|11\rangle)_{23}\\
&+\Psi^+_{b1}\otimes(\beta|00\rangle+\alpha|11\rangle)_{23}+\Psi^-_{b1}\otimes(\beta|00\rangle-\alpha|11\rangle)_{23}\}
\end{aligned}
\tag{7}
$$

and

$$
\begin{aligned}
&\frac{1}{2\sqrt{2}}|-\rangle_a\otimes\{\Phi^+_{b1}\otimes(\alpha|00\rangle-\beta|11\rangle)_{23}-\Phi^-_{b1}\otimes(\alpha|00\rangle+\beta|11\rangle)_{23}\\
&+\Psi^+_{b1}\otimes(-\beta|00\rangle+\alpha|11\rangle)_{23}-\Psi^-_{b1}\otimes(\beta|00\rangle-\alpha|11\rangle)_{23}\}.
\end{aligned}
\tag{8}
$$

According to (7) and (8), Bob can perfectly reconstruct the pure EPR state using the corresponding unitary operations given in Table 1, which shows the unitary operations for Bob corresponding to various results of the measurements.

For example, if the $r\_m_1=|+\rangle$ and $r\_m_2=|\Phi^+\rangle$, Bob will perform the corresponding unitary operations $I\otimes\sigma_x$ to reconstruct the pure EPR state.

Table 1 The corresponding unitary operations of Bob

r_m_1	r_m_2	Operations of Bob
$\lvert+\rangle$	$\lvert\Phi^+\rangle$	$I\otimes\sigma_x$
$\lvert+\rangle$	$\lvert\Phi^-\rangle$	$I\otimes i\sigma_y$
$\lvert+\rangle$	$\lvert\Psi^+\rangle$	$\sigma_x\otimes I$
$\lvert+\rangle$	$\lvert\Psi^-\rangle$	$i\sigma_y\otimes I$
$\lvert-\rangle$	$\lvert\Phi^+\rangle$	$I\otimes i\sigma_y$
$\lvert-\rangle$	$\lvert\Phi^-\rangle$	$I\otimes\sigma_x$
$\lvert-\rangle$	$\lvert\Psi^+\rangle$	$i\sigma_y\otimes I$
$\lvert-\rangle$	$\lvert\Psi^-\rangle$	$\sigma_x\otimes I$

3 Quantum Secret State Sharing Protocol

A teleportation scheme of an unknown pure EPR state via the GHZ-like state has been proposed in Sect. 2. This section demonstrates that this teleportation scheme can be used to develop a QSSS protocol.

Suppose that a manager, Alice, wants to give two agents, Bob and Charlie, n secret quantum states $|\phi_{EPR}\rangle_{ab}^i=\alpha_i|01\rangle_{ab}+\beta_i|10\rangle_{ab}$, where the superscript i means the ith pair of the n secret quantum states and $i=1$ to n. The procedure of the proposed QSSS protocol is explained in the following (see also Fig. 2).

Step 1. Alice prepares $2n$ GHZ-like states $|\phi_G\rangle_{123}^j$, keeps the first particle q_1^j of each GHZ-like states to herself, and then distributes the second and the third particles (q_2^j, q_3^j) of each GHZ-like states to Bob and Charlie respectively, where $j=1$ to $2n$.

Step 2. After receiving these particles, Alice, Bob, and Charlie use a half of particles to do the eavesdropping detection (e.g., random sampling discussion). If the error rate of the public discussion exceeds a preset threshold, this communication will be aborted. Otherwise, she continues the following steps.

Step 3. Alice performs $q_b^i\otimes q_1^i$, measures the q_a^i by D-basis $\{|+\rangle,|-\rangle\}$, and then performs the Bell-measure on q_b^i and q_1^i. Let the measuring results be denoted as $r\_m_1^i$ and $r\_m_2^i$, respectively. Then, she announces $r\_m_1^i$ and $r\_m_2^i$ to Bob and Charlie.

Step 4. When cooperating together, Bob and Charlie performs U_S and the corresponding operations on the q_2^i and q_3^i. Finally, the secret quantum state $|\phi_{EPR}\rangle_{ab}^i$ can be recovered accordingly.

According to [6], the communicating information using the quantum teleportation is as secure as the one-time pad. A quantum teleportation can be seen as the fully quantum version of the one-time pad. However, a quantum teleportation can only be correctly executed under a genuine entanglement shared between a sender and a receiver. Therefore, an enough large subset (which is n GHZ-like states) from these $2n$ GHZ-like states is used to check the eavesdropping on the quantum channel in Step 2. In this case, an eavesdropper escapes from the eavesdropping check with only a negligible probability; that is, the remaining tripartite entanglement particles of the GHZ-like state are not interfered and the correct correlations of entanglements can be ensured. Thus, Alice's secret quantum state can be faithfully and securely transferred to Bob and Charlie without any information leakage to the outsider. Consequently, the security of transmitting secret particle states is as secure as the quantum one-time pad in the proposed QSSS protocol.

Furthermore, the $q_2^i(q_3^i)$ held by Bob (Charlie) is only one particle of the ith secure entangled state so that no information is leaked. In other words, no information of a secret

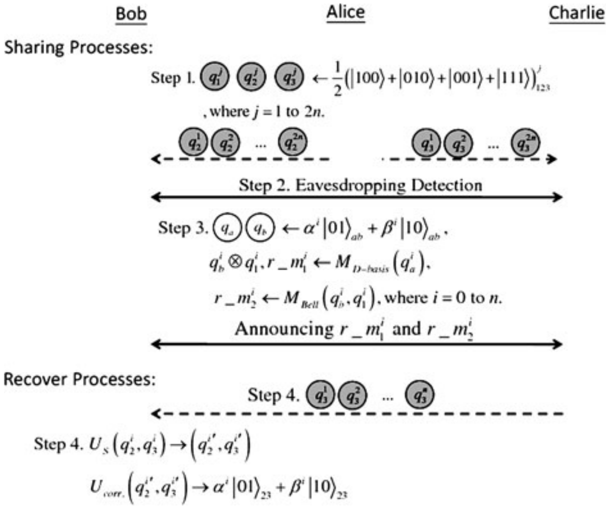

Fig. 2 The proposed QSS protocol of quantum states

quantum state is obtained by Bob (Charlie). Therefore, Bob and Charlie must cooperate with each other to recover the secret quantum state of Alice.

4 Conclusion

This study solves the open problem proposed by Yang et al. in [13] and shows that the GHZ-like state can be used to realize the teleportation scheme of a pure EPR state. Since the EPR state and controlled-not gate can be implemented in practice, the experimental development of the proposed teleportation scheme is practical. Besides, a QSSS protocol is proposed using this teleportation scheme. Though this paper successfully teleports an EPR state via the GHZ-like state, is it possible to teleport more than two-particle state via the GHZ-like state? It indeed is an interesting research topic in the future.

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